SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2006
YEAR 11 MATHEMATICS EXTENSION
HSC Task #1

Mathematics Extension

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Start each new question in a separate answer booklet.
- Marks may **NOT** be awarded for untidy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 76

Examiner: *D.McQuillan*

Total marks – 76
All question are NOT of equal value

Answer each QUESTION in a SEPARATE writing booklet

Question One

Marks

(a) Which of the following are NOT polynomials. **3**

(A) $x\sqrt{x}+2x$ **(B)** $\sqrt{2}x^3+5x-2$ **(C)** $2x^3+5x^{-2}$

(D) -5 **(E)** $3+7x^3-2x^5$ **(F)** $\dfrac{x^2-4}{x^2-x-2}$.

(b) For the parabola with focus (-1, 4) and vertex (-1, 0) find: **3**

(i) the focal length

(ii) equation of the directrix

(iii) equation of the parabola

(c) Show that $x+1$ is a factor of x^3-5x^2+3x+9. **2**

(d) **3**

(i) Write down the expansion of $\sin(A+B)$.

(ii) Hence find the exact value of $\sin(105°)$.

(e) If $\tan\alpha=\dfrac{2}{3}$ and $\tan\beta=\dfrac{1}{5}$ find the value of $\tan(\alpha-\beta)$. **2**

(f) Given two polynomials, $P(x)$ of degree m, $Q(x)$ of degree n and $\frac{m}{n} > 1$. Find the degree of: **2**

(i) $P(x) \times Q(x)$

(ii) $Q(x) - P(x)$

(g) Simplify $\sin A \sin 2A + \cos A \cos 2A$. **2**

(h) By expressing $6x^3 + 25x^2 + 5x - 1$ in the form $(3x+2)Q(x) + R(x)$ find the polynomials $Q(x)$ and $R(x)$. Given that $R(x)$ is a constant. **2**

End of Question One

START A NEW WRITING BOOKLET

Question Two

Marks

(a) Sketch the graphs of the following equations showing all x and y-intercepts. **4**

(i) $y=(x-1)^3$

(ii) $y=(x+4)^2(x-1)(5-x)$

(b) The circles with centres (-3, -2) and (5, 4) and radii 7 and 3 respectively, intersect at only one point. By considering the division of an interval into a ratio find the point of intersection. **2**

(c) When $x^4-8x^3-mx^2-(m-4)x-27$ is divided by $x-2$ the remainder is –7. Find the value of m. **2**

(d) Find an equation of a line with an x-intercept of 3 and makes an angle of 45° with the line $y=2x+5$. **2**

(e) Find the Cartesian equation of the parabola with parametric equations $x=\frac{t}{2}+5$, $y=t^2-1$. **2**

(f) The roots of $2x^3 + 6x + 3 = 0$ are α, β, γ. **4**

Find the value of:

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

(ii) $\alpha^2 + \beta^2 + \gamma^2$

(g) Express $5x^3 - 15x^2 + 13x - 1$ in the form $A(x-1)^3 + B(x-1) + C$ **3**

End of Question Two

START A NEW WRITING BOOKLET

Question Three

Marks

(a) Jamie and Adam want to secure a vertical pole by attaching guy wires to the top. The pole is due north of Adam and the angle of elevation to the top is $15^{o}18'$, this point is where the first wire will be grounded. Adam now walks 68m due west and finds the pole has a bearing 067^{o}, this point is where a second wire is to grounded. Find the length of wire they will need for these two wires, rounded up to the nearest metre. **3**

(b) The points $P\left(6p,3p^2\right)$ and $Q\left(6q,3q^2\right)$ lie on the parabola $x^2=12y$. **6**

(i) Show that the equation of the tangent at P is $y=px-3p^2$.

(ii) Hence write down the equation of the tangent at Q.

(iii) Find the point of intersection of the two tangents.

(c) Prove the identity $\dfrac{\sin\phi}{1-\cos\phi}=\operatorname{cosec}\phi+\cot\phi$. **3**

(d) In a trapezium *ABCD*, *AB* is parallel to *DC*, *AB* = *BC* = 10cm, *CD* = 7cm and *DA* = 8cm. Find the size of $\angle BCD$ to the nearest minute. **2**

(e) Given the expression $6\sin x + 2\sqrt{3}\cos x$. **6**

(i) Write the expression in the form $R\sin(x+\alpha)$ where $R > 0$ and α is acute.

(ii) Hence solve $6\sin x + 2\sqrt{3}\cos x = 2\sqrt{6}$, for $0 \le x \le 2\pi$.

(iii) Write down the general solution for the equation in part (ii).

(iv) Find the greatest value of $6\sin x + 2\sqrt{3}\cos x$.

End of Question Three

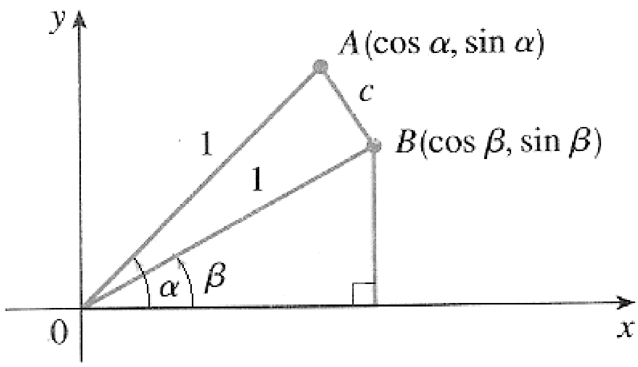
START A NEW WRITING BOOKLET

Question Four

Marks

(a) Show that $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)=\sec\theta+\tan\theta$. **3**

(b) Given the figure, **6**



(i) Prove the subtraction formula

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

by comparing two expressions for c^2.

(ii) Using part (i) and the fact that $\cos\theta$ is an even function and $\sin\theta$ is an odd function, deduce the addition formula for cosine.

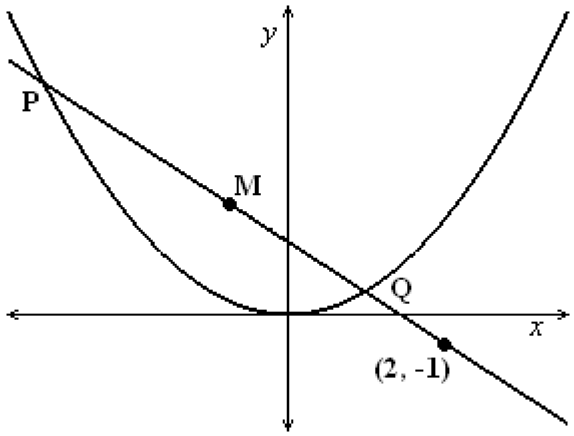
(iii) Use the addition formula for cosine and the identities

$$\cos(90°-\theta)=\sin\theta \qquad \sin(90°-\theta)=\cos\theta$$

to prove the subtraction formula for the sine function.

(c) Solve $\tan x \sin x = 3\sin x - \sec x$, for $0° \le x \le 180°$.
Round to the nearest minute. **3**

(d) The points $P\left(p, \frac{p^2}{2}\right)$ and $Q\left(q, \frac{q^2}{2}\right)$ lie on the parabola $x^2 = 2y$. **6**



(i) Show that the equation of the chord PQ is

$$y = \frac{(p+q)x - pq}{2}$$

(ii) If PQ passes through (2, -1) show that $pq = 2p + 2q + 2$.

(iii) Find the locus of M the mid-point of PQ.

End of Question Four

End of Paper

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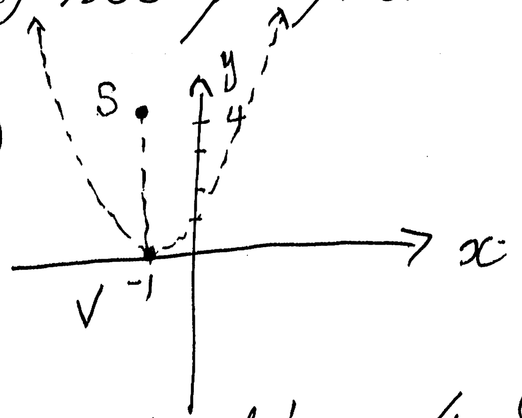
Q1/ (a) not polynomials A, C, F. (3)

(b)



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(i) focal length 'a' = 4 (1)

(ii) eq^n directrix $y = -4$ (1)

(iii) eq^n parabola $(x-h)^2 = 4a(y-k)$.

$(h,k) = (-1, 0)$ $\quad (x+1)^2 = 4 \times 4(y-0)$

$(x+1)^2 = 16y$. (1)

(c) could use long division

or let $P(x) = x^3 - 5x^2 + 3x + 9$

let $x+1=0 \Rightarrow x = -1$.

So $P(-1) = -1^3 - 5\times -1^2 + 3\times -1 + 9$

$= -1 - 5 - 3 + 9 = 0$ Is a factor! (2)

(d) (i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ (1)

(ii) $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$ (2)

(e) $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$= \dfrac{\frac{2}{3} - \frac{1}{5}}{1+\frac{2}{3}\times\frac{1}{5}} = \dfrac{\frac{7}{15}}{1\frac{2}{15}} = \dfrac{7}{17}$ (2)

(f) $P(x)$ degree m
$Q(x)$ degree n

$\dfrac{m}{n} > 1 \Rightarrow m > n$

(i) add indices $m+n$ (1)
(ii) bigger index stays m (1)

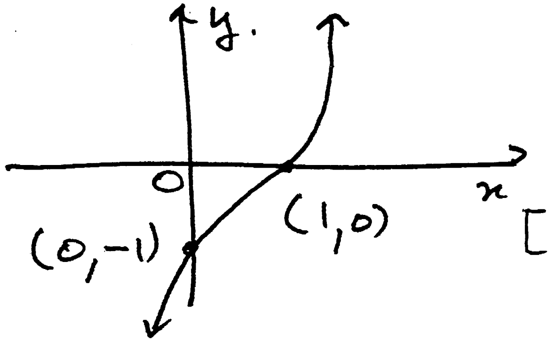
(g) $\cos(x-y) = \cos x\cos y + \sin x \sin y$
So $\cos A\cos 2A + \sin A\sin 2A$
$= \cos(A-2A)$
$= \cos(-A)$
$= \cos A$ even fn (2)

(h)
$$\begin{array}{r l}
 & 2x^2 + 7x - 3 \\
3x+2 \,\big)\!\! & \overline{6x^3 + 25x^2 + 5x - 1} \quad - \\
 & 6x^3 + 4x^2 \\
\hline
 & 21x^2 + 5x - 1 \quad - \\
 & 21x^2 + 14x \\
\hline
 & -9x - 1 \quad - \\
 & -9x - 6 \\
\hline
 & 5
\end{array}$$

$(3x+2)(2x^2+7x-3)+5$ (2)

Solution Question [2]

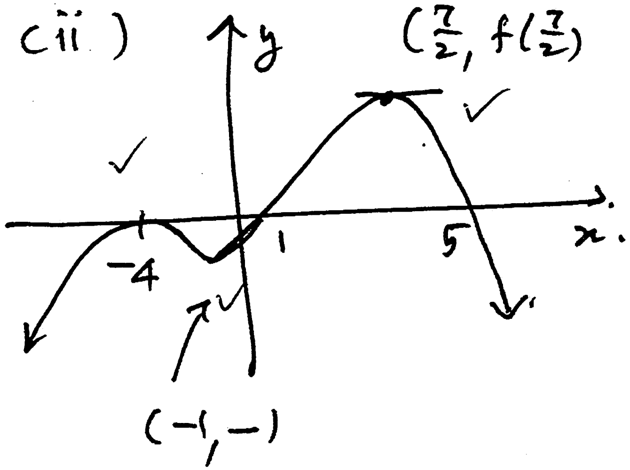
• (a) (i) $y = (x-1)^3$



[1]

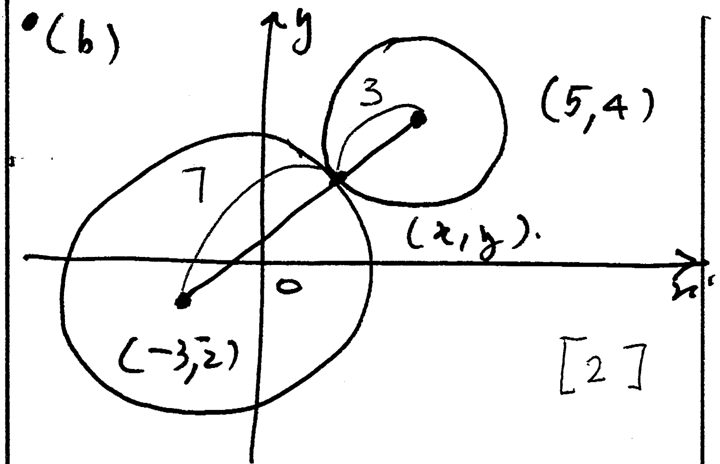
$x = 0, \quad y = -1.$

(ii)



[3]

• (b)



[2]

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{7 \times 5 + (3 \times -3)}{10}$$

$$= \frac{35 - 9}{10}$$

$2\frac{3}{5} = 13/5 \quad = 2.6$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{7 \times 4 + (3 \times -2)}{10}$$

$2\frac{1}{5} = 11/5 = \quad 2.2$

∴ The pt is

$(2.6, 2.2)$.

• (c) $P(x) = x^4 - 8x^3 - mx^2 - (m-4)x - 27.$

$P(2) = -7.$ [2]

$P(2) = 16 - 64 - 4m - 2m + (-19$

∴ $-6m - 67 = -7$

$-6m = 60, \quad m = -10$

• (d) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$

Now $1 = \left|\frac{m-2}{1+2m}\right|$

∴ $|1 + 2m| = |m - 2|$ [2]

$1 + 2m = m - 2$ or $1 + 2m = 2 - m$

$m = -3.$

$3m = 1$

$m = \frac{1}{3}$

$y = -3x + b$

$0 = -9 + b$

∴ $b = 9$

∴ $y = \frac{x}{3} + b.$

• $y = -3x + 9$

$3x + y - 9 = 0$

$y = \frac{x}{3} - 1$

• $x - 3y - 3 = 0$

• (e) $x = \frac{t}{2} + 5$

$\therefore t = 2(x-5)$ —— 1

$y = t^2 - 1$ [2]

$\therefore \boxed{y = 4(x-5)^2 - 1.}$

or $= 4(x^2 - 10x + 25) - 1$

$\boxed{y = 4x^2 - 40x + 99}$

• (f) $2x^3 + 6x + 3 = 0$ [4]

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{3}{-3/2}$ 2

$= -2.$

(ii) $(\alpha^2+\beta^2+\gamma^2) = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 0 - 2(3)$ 2

$= -6.$

• (g) $5x^3 - 15x^2 + 13x - 1 \equiv A(x-1)^3 + B(x-1) + C.$

(i) Equate coefficient of x^3 [3]

$\therefore \boxed{A = 5}$ 1

(ii) Put $x = 1$,

$5 - 15 + 12 = C$ $\therefore \boxed{C = 2}$ 1

(iii) put $x = 0$

$-A - B + C = -1$

$-5 - B + 2 = -1$

$\boxed{B = -2}$ 1

$\therefore 5x^3 - 15x^2 + 13x - 1$

$\boxed{= 5(x-1)^3 - 2(x-1) + 2.}$

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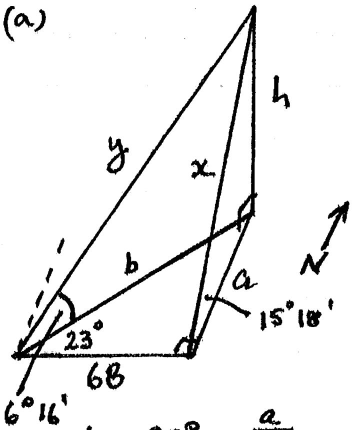
Question 3

(a)



$\tan 23° = \frac{a}{68}$

$a = 68\tan 23°$

$\cos 15°18' = \frac{a}{x}$

$\therefore x = \frac{a}{\cos 15°18'}$

$= \frac{68\tan 23°}{\cos 15°18'}$

$= 29.9249\ldots$

$\cos 23° = \frac{68}{b}$

$b = \frac{68}{\cos 23°}$

$\cos 6°16' = \frac{b}{y}$

$y = \frac{b}{\cos 6°16'}$

$= \frac{68}{\cos 23° \cos 6°16'}$

$= 74.3165\ldots$ 3

$\therefore x + y = 104.241\ldots$

$\approx 105\,m$ (rounded up)

(b) (i) $y = \frac{1}{12}x^2$

$y' = \frac{1}{6}x$

At P, eqn of tangent is

$y - 3p^2 = p(x - 6p)$

$y - 3p^2 = px - 6p^2$ 3

$y = px - 3p^2$

(ii) $y = qx - 3q^2$ 1

(iii) $px - 3p^2 = qx - 3q^2$

$(p-q)x = 3p^2 - 3q^2$

$(p-q)x = 3(p-q)(p+q)$

$x = 3(p+q)$ 2

$y = p(3(p+q)) - 3p^2$

$= 3pq$ 6

Pt of $\cap$ is $(3(p+q), 3pq)$

(c) RHS $= \frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}$

$= \frac{1+\cos\phi}{\sin\phi}$

$= \frac{(1+\cos\phi)(1-\cos\phi)}{\sin\phi(1-\cos\phi)}$

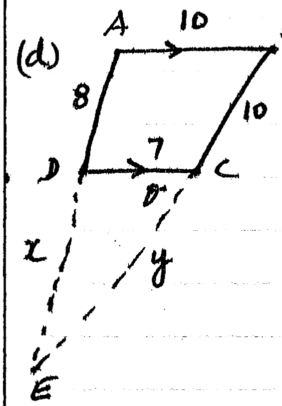
$= \frac{1-\cos^2\phi}{\sin\phi(1-\cos\phi)}$

$= \frac{\sin^2\phi}{\sin\phi(1-\cos\phi)}$ 3

$= \frac{\sin\phi}{1-\cos\phi}$

= LHS as required.

(d)



$\frac{x}{x+8} = \frac{y}{y+10} = \frac{7}{10}$

$10x = 7x + 56$

$3x = 56$

$x = \frac{56}{3}$

$10y = 7y + 70$

$3y = 70$

$y = \frac{70}{3}$

$\cos\theta = \frac{7^2 + \left(\frac{70}{3}\right)^2 - \left(\frac{56}{3}\right)^2}{2\times 7\times\frac{70}{3}}$

$= \frac{3}{4}$

$\therefore \theta = 41°25'$ 2

$\therefore \angle BCD = 138°35'$

(e) (i) $6\sin x + 2\sqrt{3}\cos x$

$= 4\sqrt{3}\left(\frac{6}{4\sqrt{3}}\sin x + \frac{2\sqrt{3}}{4\sqrt{3}}\cos x\right)$

$= 4\sqrt{3}\left(\sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2}\right)$

$= 4\sqrt{3}\left(\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right)$

$= 4\sqrt{3}\sin\left(x + \frac{\pi}{6}\right)$ 2

(ii) $4\sqrt{3}\sin\left(x+\frac{\pi}{6}\right) = 2\sqrt{6}$

$\sin\left(x+\frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$

$x + \frac{\pi}{6} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ 2

$x = \frac{\pi}{12}$ or $\frac{7\pi}{12}$

(iii) General solutions $\frac{\pi}{12} + 2n\pi$ 1

$\frac{7\pi}{12} + 2n\pi$

(iv) Max value when $\sin\left(x+\frac{\pi}{6}\right) = 1$ 1

$\therefore$ Max value $= 4\sqrt{3}$ 6

Yr 11 Ext1 Task #1 (06)

Question 4

(a) LHS $= \dfrac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{\theta}{2}}$

$= \dfrac{1+t}{1-t}$ where $t = \tan\frac{\theta}{2}$

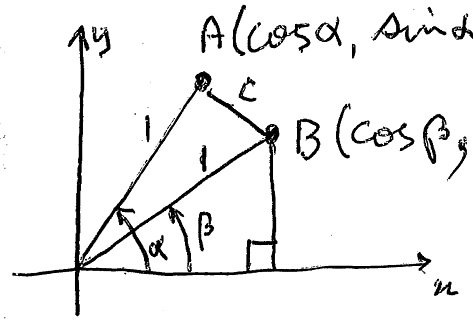
$= \dfrac{(1+t)^2}{1-t^2}$

$= \dfrac{1+t^2+2t}{1-t^2}$

$= \dfrac{1+t^2}{1-t^2} + \dfrac{2t}{1-t^2}$

$= \sec\theta + \tan\theta$

$=$ RHS [3]

b) 

(i) Distance formula

$c^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$= 2 - 2(\cos\alpha\cos\beta + \sin\alpha\cos\beta)$

By the Cosine Rule

$c^2 = 1 + 1 - 2\cos(\alpha - \beta)$

$\therefore 2 - 2\cos(\alpha-\beta) = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

[2]

(ii) Replace β with $-\beta$:

$\therefore \cos(\alpha-(-\beta)) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$

$\therefore \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(cosine is even, sine is odd) [2]

(iii) $\sin(\alpha-\beta) = \cos(90-(\alpha-\beta))$

$= \cos((90-\alpha)+\beta)$

$= \cos(90-\alpha)\cos\beta - \sin(90-\alpha)\sin\beta$

$\therefore \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

[2]

(c) $\tan x\sin x = 3\sin x - \sec x$ $(\cos x \neq 0)$

$\dfrac{\sin^2 x}{\cos x} = 3\sin x - \dfrac{1}{\cos x}$

$\therefore \sin^2 x = 3\sin x\cdot\cos x - 1$

$\therefore \sin^2 x = 3\sin x\cos x - \cos^2 x - \sin^2 x$

$\therefore 2\sin^2 x - 3\sin x\cos x + \cos^2 x = 0$

$(2\sin x - \cos x)(\sin x - \cos x) = 0$

$\therefore 2\sin x - \cos x = 0$ or $\sin x - \cos x = 0$

$\tan x = \frac{1}{2}$ or $\tan x = 1$

$x \doteqdot 26^\circ 34'$, $x = 45^\circ$ [3]

(d) $P(p, \frac{p^2}{2})$, $Q(q, \frac{q^2}{2})$ lie on $x^2 = 2y$

(i) Chord PQ:

$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$\dfrac{y-\frac{p^2}{2}}{\frac{q^2}{2}-\frac{p^2}{2}} = \dfrac{x-p}{q-p}$

$$\frac{2\left(y-\frac{p^2}{2}\right)}{q^2-p^2} = \frac{x-p}{q-p}$$

$$\therefore\ 2y-\cancel{p^2} = (x-p)(q+p)$$
$$= (p+q)x-\cancel{p^2}-pq$$

[2] $\therefore\ y = \frac{1}{2}\left((p+q)x-pq\right)$

(ii) Substitute $(2,-1)$:

$$-1 = \frac{1}{2}(2p+2q-pq)$$
$$-2 = 2p+2q-pq$$

[2] $\therefore\ pq = 2p+2q+2$

(iii) For m:

$$x = \frac{p+q}{2}\ ;\ y = \frac{\frac{p^2}{2}+\frac{q^2}{2}}{2}$$
$$= \frac{p^2+q^2}{4}$$

$\therefore\ p+q=2x \quad p^2+q^2=4y$

Now $p^2+q^2=(p+q)^2-2pq$

$$\therefore\ 4y = (2x)^2-2(2p+2q+2)$$
$$= 4x^2-2(2(2x)+2)$$
$$= 4x^2-8x-4$$

$\therefore$ Locus: $y = x^2-2x-1$

[2]